

The Yang-Mills and chiral fields in six dimensions

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Abstract

In previous work [12], we constructed an action in six dimensions using Yang-Mills fields and an auxiliary Abelian field. Here we first write down all the equations of motion and the constraints which arise from such an action. From these equations we reproduce all dynamical equations and the constraints required for self-dual tensor field theory constructed by Lambert-Papageorgakis, which describes (2,0) supersymmetric CFT in 6D. This is an indication of the fact that our 6D gauge theory contains all the same information as the on-shell theory of chiral tensor fields.

1 Introduction

Recent progress of holographic membrane theories [1]-[8], provides us with ample motivation to try and understand the 6-dimensional M5-brane theory. The latest attempts on this subject can be found in the works [9, 10, 11]. While some headways in constructing such theories having maximal supersymmetry could be found in the papers [9, 12], and the subsequent generalisations in [13]-[19]. As per the current understanding, the dynamics of single M5-brane is governed by an Abelian 6D conformal tensor theory having maximal (2,0) supersymmetry. The antisymmetric 2-rank tensor fields are natural to occur in six dimensions. There are other important dynamical reasons to include tensors in these 6D constructions. Let us take the example of an extended M2-brane ending on M5-brane. The intersection of these extended branes produces an infinitely long line defect on the world-volume of M5-brane. Such defects do constitute the simplest excitations which entirely live on the M5-brane. Basically, the defects behave like extended ‘strings’ living in a six-dimensional flat spacetime. It also makes us believe that ultimately the dynamics of these stretched string-defects will constitute the low energy dynamics of the M5-branes. We may also consider other configurations where we have N parallel (coincident) M5-branes and a single M2-brane ends on them. In that situation M2-brane will produce line defects on each single M5-brane in the stack. Thus we will have a lowest energy configuration on the stack which has to be described by N parallel (spatially aligned) strings in 6D. Of course, these ‘lowest’ energy configurations would spontaneously break the rotational symmetry on the 5-branes from $SO(5) \rightarrow SO(4)$.¹ Thus we see that low energy states (vacua) of M5-brane theory could well have manifestly broken Lorentzian symmetry. Hence it would be worth while to include auxiliary Abelian vector, η^M , in the 6D gauge theory to describe this low energy dynamics, so long as Lorentz invariant configurations (vacua) are also permitted in the theory. It is known that the v.e.v. of this auxiliary vector field will always break the Lorentz symmetry.

The dynamical strings would naturally couple to antisymmetric tensor field, B_{MN} , whose field strength $H_{(3)} = dB_{(2)}$ is a 3-form. But this field strength needs to be self-dual in order to describe M5-brane. The string like solutions living on M5-brane are already known to exist [22]. In fact, a self-dual tensor field, *five* scalars, X^I , and a Majorana-Weyl spinor, Ψ , constitute what is known as the simplest (2,0) tensor multiplet in 6-dimensions [21]. The dynamical equations of chiral tensor theory are

$$H_{(3)} \equiv dB_{(2)} = \star_6 H_{(3)}, \quad \partial_M \partial^M X^I = 0 = \not{\partial} \Psi \quad (1)$$

where \star_6 is the Hodge-dual in six dimensions. This Abelian tensor theory is superconformal, but the theory is trivial as it is not interacting. It is being currently argued that all the states of a non-abelian (2,0) tensor theory, when compactified on a circle, are perhaps contained in the 5-dimensional super-Yang-Mills (SYM) theory. As such 5D

¹The situation here may crudely be compared to the case of alignment of spins in magnetism in the low energy (temperature) states. Full rotational symmetry in these systems is obtained only in the disordered (high temperature) phase.

SYM is known to be nonrenormalizable and has a strongly coupled fixed point in the UV. But if SYM indeed contains all the states of a compactified 6D CFT without requiring new degrees of freedom at higher loops, then the SYM ought to be a finite theory in itself [10, 11]. Although intuitive, but it is a very difficult task to check the finiteness of 5D SYM. Any deviation from the expected behaviour of SYM will have direct consequences for 6D (2,0) theory, see recent attempts in this direction [20].

Although very little is known about the ‘non-Abelian’ (2,0) tensor theory, which is supposed to describe the dynamics on the stack of M5-branes, but some attempts have been made recently to write down a theory using self-dual tensors [9], and by directly uplifting 5D Super-Yang-Mills action to six-dimensions [12]. Actually, a non-Abelian 6D CFT, in a simple setting, should possess $SU(N)$ gauge symmetry and $SO(5)$ global symmetry as well as conformal symmetry. The 6D gauge action provided in [12] inherits some of these features directly from SYM, as it is a direct uplift from 5D. Nevertheless these are some of the requirements which may guide us in the construction of a meaningful M5-brane theory.²

The goal of this work is to present a 6D action involving Yang-Mills fields, and an auxiliary vector field following our earlier work [12]. We write down all the equations of motion of this theory determined by its action. We then show that these equations are the same as in the work of Lambert-papageorgakis [9], which involves an on-shell construction of (2,0) chiral tensor theory. The paper is organised as follows. In section-2, we systematically work out the equations of motions for the Abelian and non-Abelian theories and also write down the constraint equations in these theory. We then introduce self-dual tensor fields and rewrite field equations in terms of these chiral tensors. In section-3, we present some solutions of the theory. The conclusions are given in the section-4.

2 6D gauge field theories

2.1 Abelian gauge fields and chiral fields

It has been proposed recently [12] that a covariant six-dimensional gauge action (in an axial form) involving scalar fields, could be written as

$$S \equiv \int d^6x \left[-\frac{1}{12(\eta)^4} (G_{MNP})^2 - \frac{1}{2} (\partial_M X^I)^2 \right] \quad (2)$$

where G_{MNP} itself is of Chern-Simons type

$$G_{MNP} = \eta_M F_{NP} + \text{cyclic permutations of indices} \quad (3)$$

while gauge field strength $F_2 = dA_1$. The vector η^M will be taken to be constant everywhere, i.e. $d\eta = 0$, but it could be lifted to be a proper abelian field with the help of a Lagrange multiplier [12]. The X^I 's ($I = 6, 7, \dots, 10$) are five real scalar fields. Note

² See earlier developments on M5-brane in the references [23, 24, 25, 26, 27, 28, 29].

that the gauge kinetic term in the action (2) is rather of unusual type. But this axial form of gauge action helps us in working with reduced gauge degrees of freedom (namely 3 on-shell vector d.o.f.s in 6D) in this special kind of covariant theory. The equations of motion following from the above action can be written as

$$\partial_M \partial^M X^I = 0, \quad d\eta = 0 \quad (4)$$

$$\eta \wedge d \star G_3 = 0, \quad (5)$$

Since $dF_2 = d(dA) = 0$, we can also write the Bianchi identity as

$$dG_3 = 0. \quad (6)$$

In our notation \star is a Hodge-dual operation in a six-dimensional Minkowski space. The equations of motion are all covariant and directly obtainable from the action (2). Let us now consider some important contractions involving constant vector $\eta \equiv \eta_M dx^M$. It simply follows from the Bianchi, $dF = 0$, that the contraction $\eta \cdot (dF) = 0$, which means that the following gauge identities involving η contractions

$$\eta^M \partial_M F_{PQ} = 0 = \eta^M F_{MN} \quad (7)$$

shall hold good. These equations are the nontrivial constraints and would remain implicit in our theory with the Lagrangian given as in (2). Naturally, the theory will allow variety of solutions, *e.g.* string-like extended solutions, monopoles and gauge instantons [12]. One can find other solutions too. Thus, any given solution of the bosonic equations will be characterised by namely the choice of η_M , A_M and X^I . We would like to show that the above equations, although looking quite different, indeed describe a chiral field theory involving self-dual 3-form tensors too!

Self-dual tensor fields: It can be noted that we have not used any 2-rank anti-symmetric tensor field in the action (2). However, given the above set up, our next aim is to define a 3-form tensor, such that it is consistent with the above equations of motion including the constraints described above and is also (anti)self-dual in nature. Such a tensor field strength could be explicitly constructed out of η and F_2 and it is given by

$$H_3 \equiv \frac{1}{2(\eta)^2} (\eta \wedge F + \star(\eta \wedge F)). \quad (8)$$

It immediately follows from the dynamical equations (4) that H satisfies the equation

$$dH = \frac{1}{2(\eta)^2} d(\eta \wedge F + \star(\eta \wedge F)) = 0. \quad (9)$$

Thus given that η and F being nontrivial, the tensor H can always be introduced. Also by construction it will also be self-dual,

$$H = \star H. \quad (10)$$

In the next step, we invert (8) and instead write down F_2 in terms of the contractions of η and H , whence

$$F_2 = 2(\eta.H) . \quad (11)$$

From this contraction we get the identity

$$dF = 0 = d(\eta.H) \quad (12)$$

Using eq.(9) we get the constraint involving the tensor

$$\eta^M \partial_M H_{PQR} = 0 . \quad (13)$$

Actually we have taken up this exercise in order to relate our Yang-Mills field equations with those of Lambert-Papageorgakis (LP) involving self-dual tensors [9]. Indeed, the bosonic equations (4) and (9) & (13) form the basis of (2,0) tensor field theory proposed by LP. Let us recall that the LP proposal had been solely based upon equations of motion, because there wouldn't exist an action in 6D, directly involving self-dual tensors. However the gauge action (2) (albeit in the axial-form) does the needful job efficiently well. This leeway to have an action is partly attached to the presence of auxiliary vector η_M in our construction. Secondly, the action (2) employs gauge fields as fundamental dynamical entities and not the tensor fields. The tensor field H introduced in (8) in that case is merely a *composite* field.

Including fermions: So far we did not say anything about the fermionic fields. It would be interesting to include suitable fermionic fields in the action (2). Particularly, the fermionic equation required for the on-shell (2,0) supersymmetry [9] is

$$\not{\partial}\Psi = 0 . \quad (14)$$

Thus a fermionic kinetic term such as $\bar{\Psi} \not{\partial}\Psi$ needs to be added to the bosonic action (2). The Abelian action including fermions becomes

$$S[A, X^I, \Psi] \equiv \int d^6x \left[-\frac{1}{12(\eta)^4} (G_{MNP})^2 - \frac{1}{2} (\partial_M X^I)^2 + \frac{i}{2} \bar{\Psi} \not{\partial}\Psi \right] \quad (15)$$

This action was originally proposed in [12]. Importantly, as we can see here that the eqs. (4),(9), (14) as well as the constraint (13) do all follow from the action (15). These equations are those which describe on-shell (2,0) supersymmetric theory [9]. The invariance of action (15) under supersymmetry

$$\begin{aligned} \delta_s X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta_s A_M &= i\eta^N \bar{\epsilon}\Gamma_{MN}\Psi \\ \delta_s \Psi &= \frac{1}{3!} H_{MNP} \Gamma^{MNP} \epsilon + \partial_M X^I \Gamma^M \Gamma^I \epsilon \\ \delta_s \eta_M &= 0 \end{aligned} \quad (16)$$

will however require other two constraints, namely

$$\eta^M \partial_M \Psi = 0 = \eta^M \partial_M X^I . \quad (17)$$

(All spinors have 32 real components. The constant spinors in supersymmetry transformations satisfy the projection condition $\Gamma_{012345}\epsilon = \epsilon$.) These latter constraints are the reflection of the fact that, although our Lagrangian density (15) is superficially 6-dimensional, actual on-shell dynamics of the fields lives in 5-dimensional space only. We comment that the constraints (17) cannot be derived from the Abelian action (15) due to the triviality (noninteracting nature) of the theory, until unless we demand the closure of the action (15) under susy. But these constraints will indeed follow rather simply in a non-Abelian (interacting) setting next.

2.2 Non-Abelian chiral fields

In the previous Abelian example we learnt that it is possible to construct a gauge action in 6D, which reproduces the field equations of a self-dual tensor theory. We would like to see if the same thing happens in the non-Abelian theory. A 6-dimensional non-Abelian gauge action including the fermions could be written as [12]

$$\begin{aligned} S_{non-Abelian} \equiv & \int d^6x \text{Tr} \left[-\frac{1}{12\eta^4} (\eta_{[M} F_{NP])^2} - \frac{1}{2} (D_M X^I)^2 + \frac{1}{4} (\eta)^2 ([X^I, X^J])^2 \right. \\ & \left. + \frac{i}{2} \bar{\Psi} \Gamma^M D_M \Psi - \frac{1}{2} \eta_M \bar{\Psi} \Gamma^M \Gamma^I [X^I, \Psi] \right] \end{aligned} \quad (18)$$

where $F_{MN} = \partial_{[M} A_{N]} - i[A_M, A_N]$ is the Yang-Mills field strength. The scalar fields X^I 's ($I = 6, 7, 8, 9, 10$) are also in the adjoint representation of the $SU(N)$. The gauge covariant derivatives are

$$D_M X^I = \partial_M X^I - i[A_M, X^I], \quad D_M \Psi = \partial_M \Psi - i[A_M, \Psi]. \quad (19)$$

The $SU(N)$ gauge symmetry of the action (18) corresponds to the fact that there are N parallel M5-branes. The gauge transformations are

$$\begin{aligned} A_M &\rightarrow A'_M = U^{-1} A_M U - i U^{-1} \partial_M U \\ X^I &\rightarrow X'^I = U^{-1} X^I U, \quad \Psi \rightarrow \Psi' = U^{-1} \Psi U \end{aligned} \quad (20)$$

under which the action (18) remains invariant, where U is an element of $SU(N)$. We now study the equations of motion which follow from the action (18). Let us simplify our notation a bit and write the 2-form gauge field strength as

$$F_2 \equiv DA = dA - i[A, A] \quad (21)$$

where $D* = d* - i[A, *]$ is used for covariant derivative. The Bianchi identity for the Yang-Mills field is

$$DF = 0 . \quad (22)$$

Since η^M is a covariantly constant (Abelian) vector, we would have

$$\eta \wedge DF = 0, \quad \text{or} \quad D(\eta \wedge F) = 0. \quad (23)$$

Also the contraction $\eta.DF$ would then imply the following constraints

$$\eta^M D_M F_{PQ}^a = 0, \quad \eta^M F_{MQ}^a = 0 \quad (24)$$

where a runs over adjoint representation of the gauge group.

Let us switch off the fermions initially. The gauge field equations obtained from the action (18) are

$$\eta \wedge D \star (\eta \wedge F^a) - \star(\eta)^4 (X_b^I D X_c^I) f^{abc} = 0 \quad (25)$$

Combining (23) and (25), it also implies that

$$\eta \wedge D(\star \eta \wedge F^a + \eta \wedge F^a) - \star(\eta)^4 (X_b^I D X_c^I) f^{abc} = 0. \quad (26)$$

At this stage, let us introduce a non-Abelian 3-form tensor, namely

$$H_3^a \equiv \frac{1}{2(\eta)^2} (\eta \wedge F^a + \star(\eta \wedge F^a)) \quad (27)$$

in the same way as in the Abelian case. It is also self-dual by construction. By inverting (27) we can also write down F in terms of H ,

$$F^a = 2(\eta.H^a) \quad (28)$$

where we used the constraint $\eta.F = 0$. The gauge Bianchi $DF = 0$, implies that

$$D(\eta.H) = 0. \quad (29)$$

It now follows from (26) that the tensor H_3 satisfies an equation

$$\eta \wedge DH^a - \frac{1}{2} \star(\eta)^2 (X^{Ib} D X^{Ic}) f^{abc} = 0. \quad (30)$$

From here it is straight forward to check that by taking a contraction of equation (30) with η , this equation can also be rewritten as a Bianchi

$$DH^a + \frac{i}{2} \eta.(\star X^{Ib} D X^{Ic}) f^{abc} = 0 \quad (31)$$

with the constraint

$$\eta^M D_M H_{PQR}^a = 0. \quad (32)$$

As an independent check once structure constants f^{abc} vanish, i.e. for $U(1)$ case, eq.(31) immediately reduce to the Abelian theory of the last section. But in the $SU(N)$ case, eq.(30) further implies a constraint, namely

$$\eta^M D_M X^I = 0. \quad (33)$$

For convenience, in standard tensorial notation, eq.(31) would give

$$D_{[M}H_{PQR]}^a - \frac{1}{2}f^{abc}\epsilon_{MPQRNS}\eta^N X^{Ib}D^S X^{Ic} = 0. \quad (34)$$

The last equation is the same equation as obtained by Lambert-Papageorgakis, when the tri-algebra there has been reduced to an ordinary Lie-algebra. The X^I equations of motion obtained from the action (18) are

$$D \wedge \star DX^I + \star(\eta)^2[X^J, [X^I, X^J]] = 0. \quad (35)$$

Including the fermions, the field equations become

$$D_{[M}H_{PQR]} + \frac{i}{2}\epsilon_{MPQRNS}\eta^N[X^I, D^S X^I] - \frac{1}{4}\epsilon_{MPQRNS}\eta^N[\bar{\Psi}, \Gamma^S \Psi] = 0 \quad (36)$$

along with the constraint

$$\eta^M D_M H_{PQR}^a = 0. \quad (37)$$

and

$$\eta_M f_{abc} \bar{\Psi}^b \Gamma^M \Psi^c = 0. \quad (38)$$

The last fermionic constraint implies that the inner product of fermionic current with vector η_M always vanishes in the vacuum. ³

Finally, the equations of motion of X^I and Ψ are

$$D \wedge \star DX^I + \star(\eta)^2[X^J, [X^I, X^J]] + \frac{1}{2} \star [\bar{\Psi}, \not{X}^I \Psi] = 0, \quad (39)$$

$$\not{D}\Psi + i \not{X}[X^I, \Gamma^I \Psi] = 0 \quad (40)$$

repectively. Thus, what has been discussed so far follows mainly from the equations and constraints directly obtainable from the action (18). The constraint which does not seem to immediately follow from the above set of equations is

$$\eta^M D_M \Psi = 0 \quad (41)$$

However, it is not difficult to figure out that eq.(39) will be consistent only when eq.(41) is included as a constraint. To ascertain this let us act with the operator $\eta^M D_M$ on the equation (39) from the left. Using the constraint (33) we find that all terms except the fermionic term $\eta^M D_M(\eta_N[\bar{\Psi}, \Gamma^N \Gamma^I \Psi])$ do vanish. Hence for the equation (39) to be consistent, the constraint (41) must follow. In summary, we have obtained all the equations and constraints, involving self-dual tensor field, which describe (2,0) supersymmetry and these all follow from the action (18). Note that we did not require any supersymmetry arguments in the above, but whatever we have obtained in the form of equations already

³ In a given vacua, if $\eta^M = (0, 0, 0, 0, 0, \eta^5)$ is aligned to be along the x^5 direction, then the 5-th component of 6D fermionic current, namely $\langle [\bar{\Psi}, \Gamma^5 \Psi] \rangle$, would vanish! It may look weird, but it is consistent with the prospect that we would like to obtain 5D SYM theory after reduction of the 6D theory on S^1 . The 5D SYM theory does not allow any operator such as $[\bar{\Psi}, \Gamma^5 \Psi]$.

describes a maximally supersymmetric theory. The supersymmetry variations of the fields can be written in the covariant form as [12]

$$\begin{aligned}
\delta_s X^I &= i\bar{\epsilon}\Gamma^I\Psi \\
\delta_s A_M &= i\eta^N\bar{\epsilon}\Gamma_{MN}\Psi \\
\delta_s \Psi &= \frac{1}{3!}H_{MNP}\Gamma^{MNP}\epsilon + D_M X^I\Gamma^M\Gamma^I\epsilon - \frac{i}{2}\eta_M[X^I, X^J]\Gamma^{IJ}\Gamma^M\epsilon \\
\delta_s \eta_M &= 0.
\end{aligned} \tag{42}$$

These match with those in [9], for an ordinary Lie-algebra, if we keep in mind our definition of the self-dual tensor. There is no need to write a separate susy transformation for H_{MNP} as it can be obtained from the variation of A_M .

3 Vacuas

There exist a number of supersymmetric vacua in the 6D gauge theory, some of which have been described in [12]. Let us note that all of these solutions will have at least one isometry direction due to the nontrivial v.e.v. of η_M . It is evident from the construction of the action that there would be no stable point-like solutions in the theory. We now list some of the static vacua of the theory and find out the components of tensor H .

- Let us first consider Lorentz symmetric vacua. It corresponds to taking $\eta_M = \text{constant}$ and $X^I = u^I$, with u^I 's being $N \times N$ diagonal constant matrices [12]. The Yang-Mills fields are vanishing for these solutions. These vacua are the maximally supersymmetric configurations and describe the moduli space corresponding to N M5-branes placed on a flat 5-dimensional transverse space. However, there exists an unique $(\eta)^2 \rightarrow 0$ limit of these solutions, such that when this limit is taken, the vacua will also preserve full $SO(1, 5)$ Lorentz symmetry of the theory.⁴ These are the only vacuas which admit full Lorentzian symmetry.
- We next consider solitonic configurations describing an extended M2-brane ending on M5-brane [12]. Consider the vacuum where $\eta^M = (0, 0, 0, 0, 0, g)$, aligned along x^5 , which we take to be an isometry direction. That is the soliton (string) is aligned along x^5 . This configuration is

$$\begin{aligned}
X^I(x^m) &= \delta^{I10}\phi(x^m), \quad (I = 6, 7, 8, 9, 10) \\
F_{0m} &= \pm g\partial_m\phi.
\end{aligned} \tag{43}$$

This configuration is a solution of equations (4) provided

$$\phi(x^m) = \phi_0 + \sum_{i=1}^p \frac{2q_i}{|x - \zeta^i|^2} \tag{44}$$

⁴ As it is clear from the actions (15) and (18) that these actions could also be written in terms of inverse vector $\xi^M = \frac{\eta^M}{(\eta)^2}$. In that case we should be taking the limit, $(\xi)^2 \rightarrow \infty$.

where fields depend upon world-volume coordinates x^m ($m = 1, 2, 3, 4$) except x^5 . Here ϕ_0 is an arbitrary constant, while ζ^i, q_i are the parameters such as positions and charges of the p solitons. The supersymmetry is preserved when

$$(1 \mp \Gamma^0 \Gamma^5 \Gamma^{10})\epsilon = 0 \quad (45)$$

Since only one of the scalar fields, namely X^{10} , representing a transverse coordinate, x^{10} , has been excited, we have a description in which M2-brane, extending along x^5 - x^{10} plane, ends on the M5-brane. The intersection is along the common direction x^5 . Such a solitonic excitation (the intersection) will create a one-dimensional string defect on M5 world-volume. The electric field surrounding the string, $E_m \equiv F_{0m}$, will be peaked near its location at x_o^i . For this solution we can now calculate the nonvanishing components of the 3-rank tensor, using (8),

$$H_{50m} = \frac{1}{2}\partial_m\phi, \quad H_{mnp} = \frac{1}{2}\epsilon_{mnp\ell 50}G^{50\ell} = \frac{1}{2}\epsilon_{mnp\ell}\partial_\ell\phi \quad (46)$$

where $\epsilon_{mnp\ell}$ is Levi-Civita tensor in four dimensions. It shows that H is self-dual.

- We next consider a magnetic monopole configuration [22]. We take η^M aligned along x^5 , as above, but we consider x^4 to be another isometry direction. We denote the remaining three spatial coordinates by x^a , with index $a = 1, 2, 3$. Over this 3-dimensional Euclidean sub-space we have a magnetic monopole solution given by

$$F_{ab} = \mp g\epsilon_{abc}\partial_c\phi, \quad X^{10}(x^a) = \phi(x^a) = \phi_0 + \sum_i \frac{2p_i}{|x - \zeta^i|} \quad (47)$$

which solves all the equations of motion in (4). For the supersymmetry variations to vanish we require following condition on the constant spinors

$$(1 \pm \Gamma^4 \Gamma^0 \Gamma^{10})\epsilon = 0. \quad (48)$$

Thus the 6D Abelian gauge theory admits $\frac{1}{2}$ -BPS monopole like solutions [12].

Correspondingly an electric type solution living over this 3-dimensional Euclidean sub-space is simply

$$F_{0a} = \mp g\partial_a\phi, \quad X^{10}(x^a) = \phi(x^a) = \phi_0 + \sum_i \frac{2q_i}{|x - \zeta^i|} \quad (49)$$

where we instead took $\eta_M = (0, 0, 0, 0, g, 0)$, i.e. here 4th component of η is nonvanishing. In this case, for the supersymmetry we still require

$$(1 \pm \Gamma^4 \Gamma^0 \Gamma^{10})\epsilon = 0. \quad (50)$$

This suggests that, if the (2,0) theory is compactified on T^2 , these electric and magnetic solutions of (49) & (47) would map into each other under the S-duality of 4D SYM theory, provided that

$$\eta_4 \leftrightarrow \eta_5.$$

It means that two sides of T^2 over which (2,0) gauge theory is compactified gets exchanged when we implement 4D S-duality. This establishes the conclusions in [10].

A mixed electro-magnetic solutions can also be found if we let η_M to be a generic vector living on T^2 , spanning (x^4, x^5) . The gauge field strength, F , should be taken to have mixed components, (F_e, F_m) , over rest of the coordinates patch (x^0, x^1, x^2, x^3) . The amount of supersymmetry will depend upon the choice of various parameters like the charges.

- Interesting instantonic solutions are found when we take η_M to be a vector having components only along, x^0 and x^5 . We shall again take $\eta_M = (0, 0, 0, 0, 0, g)$ for simplicity, as a boost can generate other component η_0 . The gauge field strength F is taken to be Yang-Mills self-dual 2-forms living over the Euclidean patch (x^1, x^2, x^3, x^4) . Accordingly the H -tensor will be

$$H_3 = \frac{1}{2g} dx^+ \wedge (F_2 + \star_4 F_2) \quad (51)$$

where $x^\pm = (x^0 \pm x^5)/2$. We see that H is definitely self-dual and satisfies $dH = 0 = d \star H$. All X^I 's are taken constant diagonal matrices [12].

4 Conclusion

We have explicitly shown that the equations and the constraints which follow from 6-dimensional gauge field action are the same as the ‘on-shell construction’ of (2,0) supersymmetric chiral tensor theory by Lambert-Papageorgakis. The important point to note is that all these equations follow from covariant 6D gauge action, in which the algebra is taken to be an ordinary Lie-algebra, for simplicity. We have demonstrated that (anti)self-dual tensors can always be introduced in our equations of motion with out requirement of any additional fields or any new algebraic structure, such as tri-algebra. However, there would always exist generic extensions of such theories to include tri-Lie-algebra [9]. In an interesting development, the authors in [13] recently presented a (1,0) supersymmetric Lagrangian theory in six dimensions. Thus it would be worth while to check if our 6D gauge action could be embedded into some reduction of the (1,0) supersymmetric theory.

Acknowledgements:

I would like to thank Anirban Basu, Neil Lambert, Shiraz Minwalla, Sunil Mukhi and especially Henning Samtleben for helpful discussions and for reading the draft. I am also grateful to the organisers of the Isaac Newton Institute, Cambridge workshop programme- ‘Mathematics and Applications of Branes in String and M-theory’ for the kind hospitality where part of this work was carried out.

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